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Bounds to the size of halo nuclei

M. Lassaut, R.J. Lombard

Groupe de Physique Théorique, Institut de Physique Nucléaire, F-91406 Orsay Cedex, France

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Abstract. Inspired by the Bertlmann-Martin inequality relating the rms radius of the ground state wave function to the lowest dipole transition energy, we have proposed a dimensional relationship to be used in weakly bound two-body systems. In the present work, it is applied to halo nuclei. Lower and upper bounds to the size of halo nuclei are compared to values obtained from reaction cross sections. The case of the deuteron is also presented.

PACS. 21.10.-K Properties of nuclei; nuclear energy level – 21.10.Gv Mass and neutron distributions

The problem of the size of halo nuclei is currently debated. Experimental values are obtained essentially from total reaction cross sections (see for instance [1, 2]). Their analysis, however, is subject to controversy [3], being sensitive to the model used to describe the reaction mechanism. Parallel momentum distributions are also available from breakup reactions,and are considered as a good way of testing the halo wave functions [4].

In the present work, we would like to discuss estimates based on general properties of weakly bound two-body systems. Our method is somewhat complementary to the earlier study of Fedorov, Jensen and Riisager [5]. The starting point is an inequality derived long ago by Bertlmann and Martin [6], valid for a particle moving in a central potential

$$\langle r^2\rangle_0 \leq \frac{3\hbar^2}{2m}\frac{1}{(E_1 - E_0)}, \tag{1}$$

where $\langle r^2\rangle_0$ is the rms radius of the ground state wave function, m the mass of the particle, E_0 the ground state energy and E_1 the lowest dipole excitation energy. The coordinate r represents the distance between the particle and the centre of the potential. This inequality is practically saturated for well bound particles [7]. The situation is quite different for weakly bound systems. On the other hand, a very loosely bound particle is not expected to be sensitive to details of the potential. Motivated by this remark, in a recent paper [8], we have studied the dimensional relationship :

$$\langle r^2\rangle_0 = \frac{3\hbar^2}{2m(-E_0)}\varphi \tag{2}$$

for an ensemble of finite range potentials admitting a single bound state.

The factor φ is obviously depending on the eigenvalue $-E_0$, as well as on the shape of the potential. However, as shown in [8], at the lower edge, where $-E_0 \to 0$, φ reaches an absolute limit, 1/6, independently of the potential. Furthermore, under specific conditions φ undergoes a kind of universal behaviour. This situation is met, in particular, for potentials having a hard core component of radius r_c followed by an attractive part. In this case we have

$$\varphi \simeq \frac{1}{6}(1 + 2\sqrt{\varepsilon} + 2\varepsilon), \tag{3}$$

where $\varepsilon = E_0/E_{0max}$, E_{0max} being the maximum possible eigenvalue such that the considered potential has a single bound state. Formula (3) is derived for large r_c. In the nuclear case, it is already valid within a few percent for $r_c \simeq 2.5$ fm. Note that the lower limit 1/6 is obtained as the interaction vanishes, independently of the presence of the hard core component.

It is very tempting to apply this relationship to halo nuclei constituted by a single neutron weakly bound to a core. We recall that in this case the separation energy is much smaller that the average binding energy per particle in the nucleus, so that a two-body description is a reasonable first order approximation. We are well aware that this simple scheme suffers from limitations. However, as long as we are interested in such a global property as the rms radius, a two-body model should be sufficient.

Thus, we assume the halo neutron to be in the 1s state of a central potential with a hard-core component of radius at least as large as the core radius. It helps to keep tract of the antisymmetrisation problem between core and halo neutrons. The dimensional relationship (2) can be used to estimate the rms radius of the halo orbital. The mass of the particle m as to be replaced by the reduced mass $\mu = \frac{m_c m_h}{(m_c+m_h)}$, where m_c and m_h denote the mass of the

Table 1. Single neutron halo nuclei. Relative distance between the halo neutron and the core nucleus. We compare the lower and the upper bounds to the experimental data. The corresponding values of φ are also given. S_n is the one neutron separation energy

	S_n [MeV]	lower bound	upper bound	r eq. (6)	φ
D	2.222	3.05	7.48	3.90 [10]	.27
				4.005 [11]	.29
^{11}Be	.504	4.76	11.65	7.00 ± .30 [3]	.36 ± .03
				6.97 ± .88 [12]	.36 ± .05
^{15}B	.97	3.39	8.30	5.26 ± 1.5 [12]	.47 ± .20
^{15}C	1.21	3.03	7.41	4.77 ± 1.40 [12]	.41 ± .12
^{17}C	.73	3.87	9.50	6.31 ± 1.3 [12]	.44 ± .25
^{19}C	.22	7.04	17.25	< 11.94 [12]	< .48
^{8}B	.1375	9.28	22.74	4.06 ± .49 [3]	.03 ± .01

core and the halo particle, respectively. The ground state eigenvalue $-E_0$ is identified with the separation energy S.

Unfortunately, knowing S does not give the value of ε; there is no universal curve relating ε and S. In the absence of any other information, we may still get from (2) a lower bound, which is independent of the potential, as stated before, and reads

$$\langle r^2\rangle_0 \geq \frac{\hbar^2}{4m}\frac{1}{S}. \tag{4}$$

At the other extreme, when the separation energy reaches the threshold for the first p state to be bound, the original Bertlmann-Martin inequality [6] gives an absolute upper bound ($\varphi(1) = 1$). At $\varepsilon = 1$ equation (3) gives a lower value, and the spreading due to different potentials yields little improvement. These two limits are too far away to get precise information on the size of halo nuclei.

On the other hand, from the measured rms radius of the halo wave function, inverting (2) provides us with an estimate of φ. The dispersion among the known cases and the variation with S should tell us about possible common features of halo nuclei.

As far as single neutron halo nuclei are concerned, two cases are relatively well measured, ^{2}H and ^{11}Be. In the former, we ignore the bound $1/2^-$ state, which is of a totally different nature, closer to the "natural" shell model state (see for instance [9]). This is certainly a limitation. However, in case of spin-orbit splitting, the transition energy appearing in the Bertlmann-Martin inequality is a weighted average of the two spin-orbit partners [7]. Since in ^{11}Be the lowest $3/2^-$ state exists as a resonance at $\approx$ 2.7 MeV, we do not expect strong corrections from the neglect of the $1/2^-$ state.

Not necessarily falling into the category of halo nuclei, the deuteron is nevertheless a typical weakly bound two-body system. We take its rms radius extracted from experimental data by Klarsfeld et al [10], as well as the value given by the Gartenhaus-Schwartz wave function [11] for the sake of comparison. The data for ^{11}Be are taken from the recent analysis of Al-Khalili et al [3] as well as previous values given by Liatard et al [12].

We complete this sample with less accurate data : ^{15}B and odd C-isotopes, namely ^{15}C, ^{17}C and ^{19}C have been measured by Liatard et al [12]. Finally, we shall discuss the case of ^{8}B taking data from [3].

We recall that in two-body systems the mean squared radius is given by

$$\langle r_A^2\rangle = \frac{A-n}{A}\langle r_{core}^2\rangle + \frac{n}{A}\langle r_h^2\rangle, \tag{5}$$

where r_{core} is the rms radius of the core nucleus and r_h is the radius of the halo wave function. Note that r_h is measured from the total cm, whereas the Bertlmann-Martin inequality is derived for the relative distance between the halo particle and the core centre of mass. The relationship between the two quantities is given by

$$\langle r^2\rangle^{1/2} = \frac{A}{A-n}\langle r_h^2\rangle^{1/2}. \tag{6}$$

The results are displayed in table 1, where the lower and the upper bounds are quoted together with the experimental relative distance obtained from (6). At the present stage, it is premature to discuss in details the spreading of φ, in view of the large uncertainties. The need for more precise data is obvious. In particular, the case for ^{19}C is of key importance. In fact, for this halo nucleus, we can quote only an upper limit. The experimental results [12] yield $\langle r_A^2\rangle < \langle r_{core}^2\rangle$; in such a case the lower limit, according to our scheme, is given by the lower bound (4).

The significant difference of φ between the deuteron and ^{11}Be underlines the different nature of the forces acting in each case. This is more stricking if we compare the E_{0max} value derived in each case from φ and S by using (3) and $\varepsilon = S/E_{0max}$. The value of the deuteron is an order of magnitude larger than the one of ^{11}Be. Consequently, although the deuteron can be classified among the weakly bound objects, it cannot be assimilated to a halo nucleus.

The case of ^{8}B brings interesting information. The apparent violation of the lower bound emphasizes both the fact that (4) has been derived without reference to the Coulomb potential and its confining effect, and that the odd proton is not in relative s-state with respect to the core. Consequently the Bertlmann-Martin inequality and the dimensional relationship (2) are not applicable to this nucleus.

It is tempting to extend the present analysis to two-neutron halo nuclei, assuming the di-neutron to be a point particle. This could be of interest if it allows us to check the validity of the two-body approximation in this case. We are not so far yet. Taking the data from [3], we get for ^{11}Li $\varphi = 0.38 \pm 0.05$. It falls in the range of values obtained for single neutron halo nuclei. However a careful analysis of the three-body problem is required before drawing firm conclusions.

To conclude, the dimensional relationship (2) allows us to analyse the halo nuclei from general properties of two-body systems. A sufficient amount of precise experimental data is still needed to discuss a conjecture concerning a common bulk aspect of single neutron halo nuclei. The extension to two-neutron halo nuclei is not straightforward, and constitutes a future goal.

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